Theoretical Analysis for Effectiveness of Spread Spectrum for Resonant type Wireless Transfer System

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ABSTRACT: For the resonant type wireless power transfer systems (WPT system) (1), this paper clarifies theoretically for tolerance of Q factor and the effectiveness of the spread spectrum (3) for noise suppression. The resonant type enables the high efficient power transmission even in small coupling coefficient k on high Q value resonant circuits. For practical uses, it requires high stability of the Q factor and suppression of high-intensity radio noise. The high efficient mechanism of the resonant type revealed theoretically by the energy loop theory (5). This theory showed that the power is transmitted during Q-time frequency period with the storage of un-transmitted energy in one period on the neighboring magnetic field (5). Based upon the energy loop theory, this paper analyze theoretically the effectiveness of spread spectrum. It becomes clear that noise suppressions can be achieved by the spread spectrum without significant loss of efficiency since transmission efficiency is tolerant for fluctuation of oscillation when the transmitter is in the ideal resonance.

KEY WORDS: resonant type WPT, tolerance of Q factors, suppression of radio noise, spread spectrum,

1. INTRODUCTION

This paper theoretically clarifies for effectiveness of the spread spectrum (3) for noise suppression on the resonant type wireless power transfer (WPT) systems (1). The resonant type WPT system enables high efficient power transmission even in small coupling coefficient k through high Q value resonance between the receiver and the transmitter (1). On the contrary, it generate larger electromagnetic noises than the conventional electromagnetic coupling type. Therefore, noise suppression technique should be developed for practical uses of the resonant type to avoid interfering already assigned radio applications.

The spread spectrum is expected to be one of promising technique for noise suppression of the resonant type (3). However, its effectiveness has not been theoretically clarified, and therefore has been concerned significant loss of efficiency since this technique requires slight frequency shifts from resonant one.

In the energy loop theory (5) proposed in the previous paper, the key factor of high efficient power transmission of the resonant type has been theoretically clarified to not be high voltage multiplied through the resonances but to be energy transfer over a plurality of cycles through accumulation in the magnetic field of the transfer failed energy during one period. This result also shows that the loop current on coil of the transmitter is multiplied by the $\mathbf{k^2QQ_2}$ times large comparing applying current from an electric power source (where k is the coupling coefficient, $\mathbf{Q}$ is the Q factor of the transmitter, $\mathbf{Q_2}$ is the Q factor of the receiver). This results implies that the applying current dwells during $\mathbf{k^2QQ_2/f_0}$ on the WPT system (where $\mathbf{f_0}$ is the supplied current frquency).

The conventional electromagnetic theory (2) tells us that the radio noises are generated by the oscillation of the magnetic dipoles driven by the loop current on the inductors. Therefore, based on the energy loop theory (5), the cause of larger electromagnetic noises of the resonant type is the superposition of currents in the same frequency during the dwell time $\mathbf{k^2QQ_2/f_0}$. Thus, one of the noise suppression strategy is to avoid the superposition

The success or failure of the above strategy depends on the tolerance of WPT system against slight frequency shift. Fortunately, WPT system is tolerant for frequency shift of the transmitter since the $\mathbf{Q_2}$ value of the receiver is not so high comparative to that of the transmitter $\mathbf{Q_1}$. Considering the relation of load with the transmission efficiency, the $\mathbf{Q_2}$ value should be from $\mathbf{Q_1/50}$ (efficiency 96%) to $\mathbf{Q_1/100}$ (efficiency 98%). This relation allows at some extent non-conformity of the resonant frequency of both circuits.

Base on the above strategy, this paper concludes that noise suppressions can be achieved by the spread spectrum (3) without significant loss of efficiency.

2. OVER VIEW OF THEORIES

2.1 formulation with the two pair circuit [5]

Before discussion of the effectiveness of spread spectrum, this paper summarizes theories used in analysis of the resonant type.

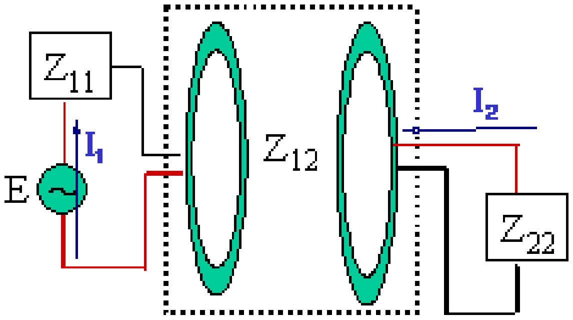

Fig. 2.1 Modeling of the WPT system

Within the $\lambda/20\pi$ rule [5] that is an engineering criterion for limit of the lump constant circuit, the WPT system can be formulated with a two pare circuit (Fig. 2.1) since phase lags of currents on circuit are negligible small for engineering purpose. Then, the Z matrix is given by

$$\begin{pmatrix} E \\ 0 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \quad (2.1)$$

In Eq. (4.1), E is the power voltage, Z_{12} is the interactions between the supplied current I_1 and the load current I_2 through the electro-magnetic fields. In the resonant type, both the transmitter and the receiver should comprise coils and capacitors. Then, $\mathbf{Z_{11}}$ and $\mathbf{Z_{22}}$ are given by

$$\mathbf{Z_{11} = R_1 + j\,\omega L_1 + 1/j\omega C_1} \quad (2.2)$$

$$\mathbf{Z_{22} = R_2 + j\,\omega L_2 + 1/j\omega C_2} \quad (2.3)$$

where $\mathbf{R_1}$ is the resistance of the coils, $\mathbf{L_1}$ is the inductance, $\mathbf{C_1}$ the capacitor of the transmitter, and $\mathbf{R_2}$ $\mathbf{(= R+R_L)}$ is the resistance of the receiver, including the load $\mathbf{R}$ and $\mathbf{R_L}$, $\mathbf{L_2}$ is the inductance of the receiver, $\mathbf{C_2}$ the capacitor of the receiver. The $\boldsymbol{\omega}$ is the frequency of power source.

Then, the supplied current I_1 and the load current I_2 are given by

$$\mathbf{I_1 = \frac{1}{R\hat{}\,(1+ k^2 Q\hat{}\,Q\hat{}_2)}\,E} \quad (2.4)$$

$$\mathbf{I_2 = \frac{-j\sqrt{k^2 Q\hat{}\,Q\hat{}_2}}{\sqrt{R\hat{}\,R\hat{}_2}\,(1 + k^2 Q\hat{}\,Q\hat{}_2)}\,E} \quad (2.5).$$

In Eq. (2.4) and (2.5), **R^, R^₂, Q^, Q^** are defined as these:

$$\mathbf{R\hat{} = R\,(1+ jQ\Omega)} \quad (2.6)$$

$$\mathbf{R\hat{}_2 = R\,(1+ jQ_2\Omega_2)} \quad (2.7)$$

$$\mathbf{Q\hat{} = \frac{Q}{(1+ jQ\Omega)}} \quad (2.8)$$

$$\mathbf{Q\hat{}_2 = \frac{Q_2}{(1+ jQ_2\Omega_2)}} \quad (2.9)$$

where Q is the Q factor of the transmitter, Q_2 is the Q factor of the receiver defined as these:

$$\mathbf{Q\hat{} = \frac{\omega_1 L}{R}} \quad (2.10)$$

$$\mathbf{Q\hat{}_2 = \frac{\omega_2 L_2}{R_2}} \quad (2.11).$$

In Eq. (2.10) and (2.11), $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega_2}$ are defined as these:

$$\mathbf{\Omega = \frac{\omega}{\omega_1} - \frac{\omega_1}{\omega}} \quad (2.12)$$

$$\mathbf{\Omega_2 = \frac{\omega}{\omega_2} - \frac{\omega_2}{\omega}} \quad (2.13)$$

where $\boldsymbol{\omega_1}$ are the natural frequency of the transmitter and $\boldsymbol{\omega_2}$ are that of the receiver. They are defined as these:

$$\mathbf{\omega_1 = \frac{1}{\sqrt{L\,C}}} \quad (2.14)$$

$$\mathbf{\omega_2 = \frac{1}{\sqrt{L_2 C_2}}}. \quad (2.15)$$

2.2 Derivation of MIT's formula [5]

The next step is to enter into resonance between the transmitter and the receiver. Under the above ideal resonance, the natural frequency of the transmitter $\boldsymbol{\omega_1}$ and that of the receiver $\boldsymbol{\omega_2}$ are equal to that of power source $\boldsymbol{\omega}$. This condition can be written as

$$\mathbf{\omega = \omega_1 = \omega_2}. \quad (2.16)$$

Then, **R^, R^₂, Q^, Q^** in Eq. (2.6), (2.7), (2.8) and (2.9) converge as these:

$$\mathbf{R\hat{} = R\,(1+ jk^2 Q\Omega) \;\rightarrow\; R}$$

$$\mathbf{R\hat{}_2 = R\,(1+ jk^2Q_2\Omega_2) \rightarrow\ R_2}$$

$$\mathbf{Q\hat{} = \frac{Q}{(1+ jk^2Q\Omega)} \rightarrow\ Q}$$

$$\mathbf{Q\hat{}_2 = \frac{Q_2}{(1+ jk^2Q_2\Omega_2)} \rightarrow\ Q_2.}$$

Therefore, we can obtain MIT's formulae of the load current such as this:

$$\mathbf{I_2 = \frac{-j\sqrt{k^2QQ_2}}{\sqrt{R\,R_2}\,(1+ k^2Q\,Q_2)}\,E} \qquad \mathbf{(2.17).}$$

The supplied current I_1 also is given by

$$\mathbf{I_1 = \frac{1}{R\,(1+ k^2QQ_2)}\,E} \qquad \mathbf{(2.18).}$$

Further, the results of Eq. (2.17) and Eq. (2.18) have been verified with the experiment [6]. In this experiment, the frequency of a power source was 100kHz of which wavelength was 3000m. Then, $\lambda/20\pi$ = 48m. The extension length of the indictor was 44.8 m. Therefore, this experiment satisfied with the $\lambda/20\pi$ rule.

2.3 energy loop on resonant circuit [5]

The two pare circuit used in the deriation of MIT's formulation is mere a phenomenologically theory. Particulaly, the load current $\mathbf{I_2}$ is greater than the supplied current $\mathbf{I_1}$. This result seems to be contradiction against the conventional common knowledge.

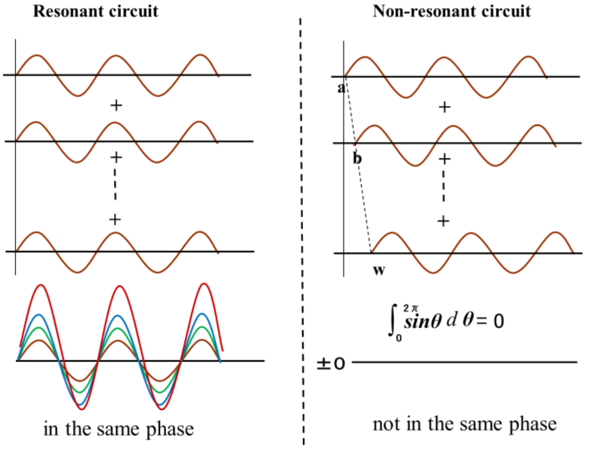

Fig. 2.2 Phase of the Secondary Currents

As illustrated in Fig 2.2, on a resonant circuit the previous current is supperposed continuously with other suceeded To obtain clear phsical view, the previous paper prposed the energy loop theory [5]. As illustrated in Fig 2.1, on a resonant circuit an current induced during one period keep in-phase for that of other periods.

currents. On other circuits, the induced current vanishs in continuous supperposition with other suceeded currents.

In the energy loop theory [5], the induced current of the power surce during one period is termed 'the primary current' and the dwelling currents after the induced period is termed 'the secondary current.'

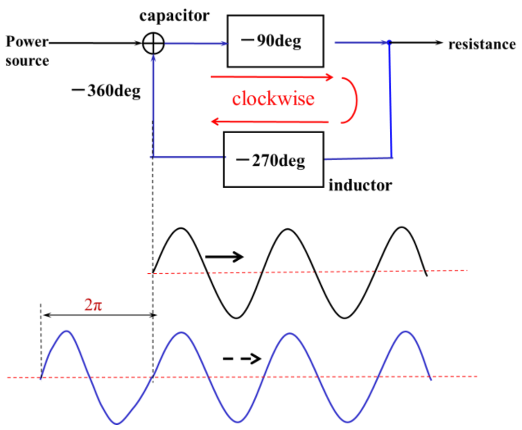

Fig. 2.3 Circulation on the Resonant Circuit

On the series resonant circuits, as illustrated in Fig 2.4, the electronic driven secondary current and the magnetic driven the secondary current circulate in mutually opposite directions. In Fig 2.4, the bold dotted lines also imply the active currents with energy and the fine dotted lines imply the inactive current lost emery. Therefore, the both secondary currents are not observable since they cancel each other on the circuit

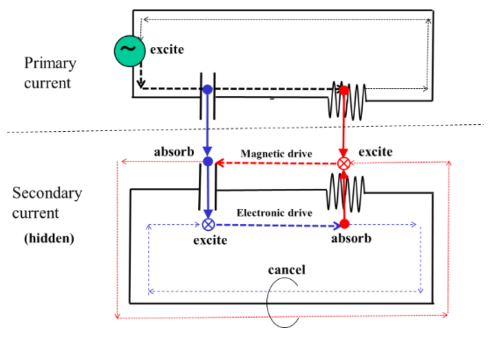

Fig.2.4 Hidden Secondary Currents

Further, in the energy loop theory [5], the attenuation constant $\boldsymbol{\tau}$ of the supplied currents is

$$\boldsymbol{\tau} \fallingdotseq \mathbf{Q/f_0} \qquad \mathbf{(2.19).}$$

where $\mathbf{f_0}$ is the resonant frequency. To say another word, the mean dwell time of the supplied currents is currents is approximately equal to

$$\mathbf{Q/f_0}. \qquad \mathbf{(2.20)}$$

Therefore, as illustrated in Eq. 2.5 the energy loop theory (5) shows that the Q times high voltage on the series resonant circuits is approximately caused by the accumulation of Q currents.

According to the above discussions, the high voltage on resonant circuits is caused by the superposition of the secondary currents

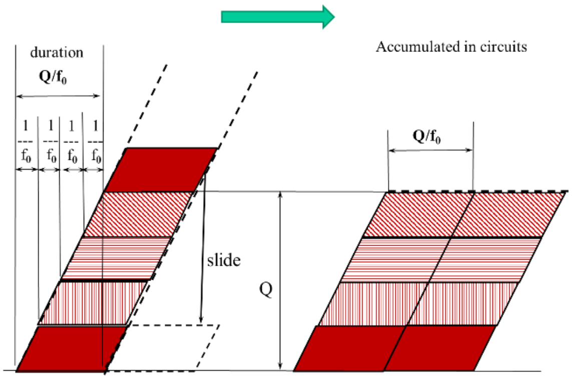

Fig. 2.5 Accumulation of Currents and Q value

For the energy flow tracing on the resonant WPT, there exist the following three energy loops

(1) Loop on the transmitter circuit
(2) Loop on the receiver circuit
(3) Echo back loop from the receiver circuit

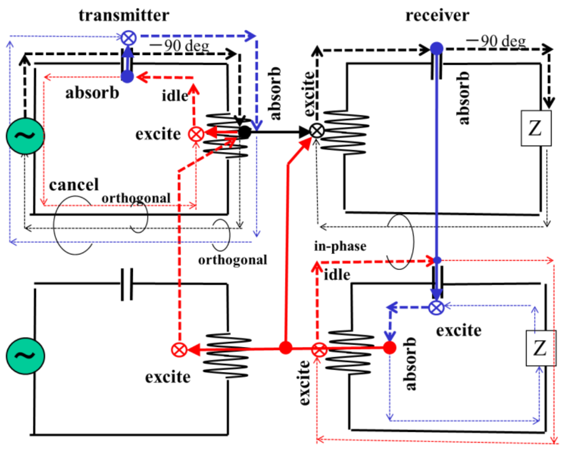

Fig. 2.6 Energy flow on the resonant type

Fig. 2.6 illustrates the above three energy loops on the resonant WPT. In Fig. 2.6, its half upper part is the transfer energy to the receiver and the loop on the transmitter circuit. Its lower part is the loop on the receiver circuit and the echo back energy from the receiver. In Fig 2.6, the bold dotted lines also imply the active currents with energy and the fine dotted lines imply the inactive current lost energy. In the resonant type, the secondary currents are not observable also since the electronic driven secondary current and the magnetic driven the secondary current cancel each other on the circuit.

From this viewpoint, these two retuned secondary currents are superposed with the primary current supplied by the power source. Therefore, high efficient energy transmission, the resonant WPT system reuses not only the energy failed to transmit but the energy failed to consume in the receiver circuit.

Based on the discussions so far, the superposition of the secondary curres is the key factor for analysis of the resonant WPT system. For the identification of the transmit inductor current $\mathbf{I_T}$ and the receiver inductor current $\mathbf{I_R}$, the previous paper sprposed the mutual escalation model (5) illustrated in Fig. 2.6.

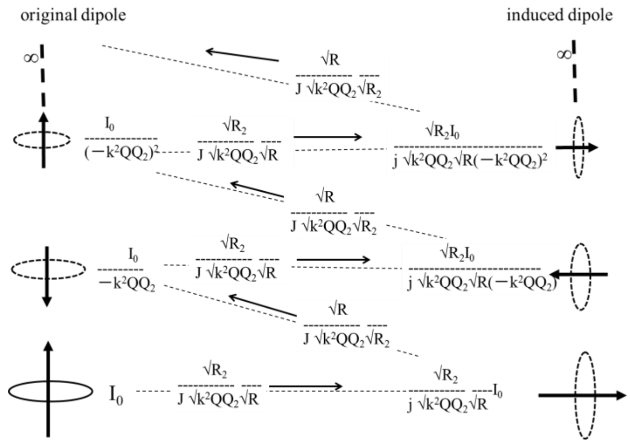

Fig. 2.7 Mutual Escalation Model

As illustrated in Fig.2.7, these recursive processes continue externally. Therefore, both the transmit inductor current $\mathbf{I_T}$ and the receiver inductor current $\mathbf{I_R}$ become as this:

$$\mathbf{I_T} = \frac{\mathbf{k^2QQ_2}}{\mathbf{R_1\,(1+k^2Q_1\,Q_2)}}\,\mathbf{E} \qquad \mathbf{(2.21)}$$

$$\mathbf{I_R} = \frac{\mathbf{-j\sqrt{k^2Q\,Q_2}}}{\mathbf{\sqrt{R\,R_2}\,(1+k^2Q\,Q_2)}}\,\mathbf{E}. \qquad \mathbf{(2.22)}$$

Comparing Eq. (2.21) with the Eq. (2.18), the transmit inductor current $\mathbf{I_T}$ is equal to the supplied current I_1 multiplied by the $\mathbf{k^2QQ_2}$ reflected with resonance between the transmitter and the receiver. Further, the receiver inductor current $\mathbf{I_R}$ is less than t the transmit inductor current $\mathbf{I_T}$. This result is satisfied with the conventional common knowledge.

3 EFFECTIVENESS OD SPRED SPECTRUM

3.1 general radiation theory for WPT system (4)

The conventional electro-magnetic theory tells us that the radio noise is generated by the oscillation of magnetic dipoles on an inductor. It is given by the monochromatic magnetic Hertz's (vector) potential $\boldsymbol{\Pi m}$ (2).

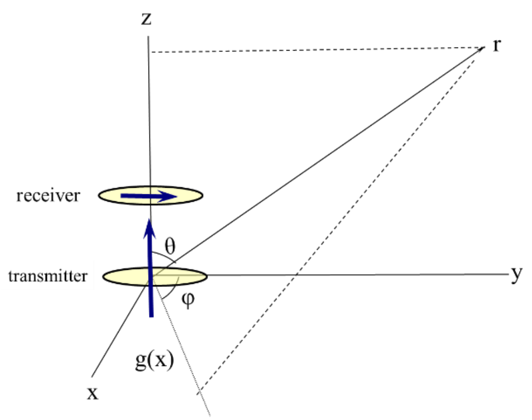

Fig. 3.1 Magnetic Dipole Moment

Generally, the line length of the coil in the WPT systems is within the λ/ 20π. The λ/20π rule (4) indicate us that phase lags on loop currents on an inductor are negligible small for engineering purpose. Therefore, the inductors in the WPT systems can be approximated with the infinitesimal loop currents.

Under the above condition, as illustrated in fig.3.1, the Hertz's (vector) potential $\boldsymbol{\Pi m}$ can be simplified with the magnetic dipole moment (4):

$$\Pi m = \frac{I\,S}{4\pi} e^{-i\omega t+ikr} \frac{1}{r} \qquad (3.1)$$

where the orientation of magnetic moment is Z axis. I is the intensity of loop current on the inductor generating magnetic dipole moment. S is the cross-section area of the loop current, ω is the angular frequency of electromagnetic wave, k is the wave number of electromagnetic wave, '**r**' is the distance from the magnetic moment.

Further, in the WPT systems, as illustrated in fig.3.2, only on the far field where r > λ, radiation becomes dominant. On this field, phase lag of radiation caused by the different path length also becomes negligible small since the gap between the transmitter and the receiver is negligible small comparing the wavelength λ. Therefore, the Hertz's (vector) potential for the WPT systems can be approximated as this:

$$\Pi m = \frac{(\mathbf{I_T} + \mathbf{I_R})\,S}{4\pi} e^{-i\omega t+ikr} \frac{1}{r} \qquad (3.2)$$

In Eq. (3.2), we must note that the above potential is independent of the power voltage.

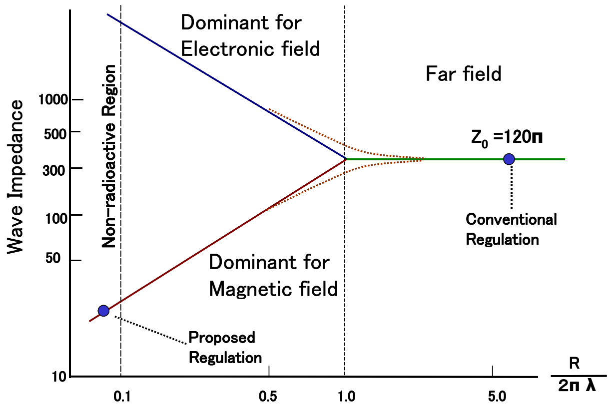

Fig. 3.2 Near and Far Fields

By differentiations of Eq. (3.2), the radiation fields is given by

$$H_\theta \rightarrow -\frac{(\mathbf{I_T} + \mathbf{I_R})\,S}{4\pi} e^{-i\omega t+ikr} \left\{\frac{k^3}{r}\right\} \sin\theta \qquad (3.3)$$

$$E_\Phi \rightarrow -Z_0 \frac{(\mathbf{I_T} + \mathbf{I_R})\,S}{4\pi} e^{-i\omega t+ikr} \left\{\frac{k^3}{r}\right\} \sin\theta \qquad (3.4)$$

where $\mathbf{H}_\theta$ is the θ component of the magnetic field, $\mathbf{E}_\Phi$ is the Φ component of the electronic field and the θ is the azimuth angle.

3.2 issue of the resonant type

As already mentioned, the electro-magnetic noise from the WPT system is proportional to the intensity of loop currents through coils. From Eq. (2.19) and Eq. (2.20), the intensity of the loop current is given by

$$\mathbf{I_T} + \mathbf{I_R}$$

$$= \left(k^2 Q_1 Q_2 - j \frac{\sqrt{R\,k^2 Q\,Q_2}}{\sqrt{R_2}} \right) \frac{1}{R\,(1+k^2 Q_1 Q_2)} \mathbf{V}. \qquad (3.5)$$

Eq. (3.5) also shows that the noise density is also multiplied by the k^2QQ_2 time large. This result also implies that the cause of large electromagnetic noises is not be high voltage but energy transfer over a plurality of cycles.

3.3 basic concept of the spread spectrum

On the resonant type, the secondary currents are superposed up to $\mathbf{k^2QQ_2/f}$ time when the power source supplied. The cause of large electromagnetic noises of the resonant type is this superposition during the dwell time $\mathbf{k^2QQ_2/f}$.

The key factor of high efficient power transmission of the resonant type is not be high voltage multiplied through the resonances but is energy transfer over a plurality of cycles through accumulation. The key to suppression of radio noises of the resonant type is to prevent superposition of the secondary currents while energy can transfer over a plurality of cycles until dwell time $\mathbf{k^2QQ_2/f}$.

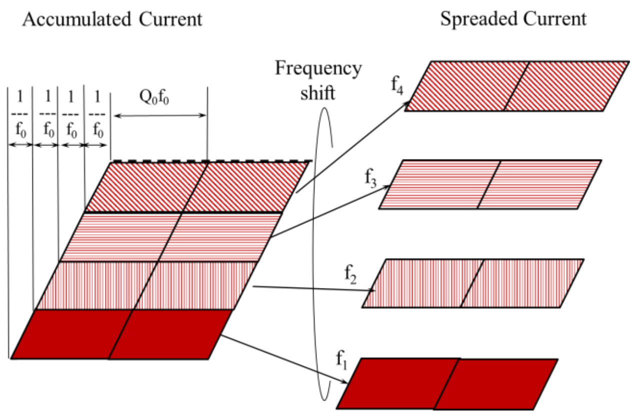

Fig3.3 frequency shifts

Based on the above concept, this paper proposes that the supplied current frequency shifts another frequency before dwell time $\mathbf{k^2QQ_2/f}$ to avoid the superposition of the secondary currents with other supplied currents. As illustrated in Fig. 3.3, after the supplied current frequency shifts another frequency, the secondary current will keep to exist without the superposition of with other supplied currents. After the dwell time $\mathbf{k^2QQ_2/f}$ at which the attenuation starts, the supplied current frequency returns the original one. In this way, the superposition of the secondary currents can be prevented while energy can transfer over a plurality of cycles. This proposed method is equivalent to the virtually parallel transmission on the frequency domain (Fig. 3.3).

In the ideal condition that all frequency are completely independent, using N frequency can reduce the superposition of the "secondary currents" to 1/N. If the supplied current frequency can be spread among $\mathbf{k^2QQ_2}\Delta f$ frequency (Δf :frequency interval), the problem of large electromagnetic noise of the resonant type completely solvable.

3.4 transfer efficiency

The success or failure of the above noises suppression strategy depends on the tolerance of transfer efficiency of the resonant type against slight frequency shift. For the resonant type, even slight frequency shifts from the resonance frequency is worried to effect significant loss of the efficiency. This section analyzes mechanism for how the transfer efficiency is determined.

The resistance of receiver R_2 comprises two parts: the resistance of coil R_C and that of load Ro. The transmission efficiency of the WPT system is defined as the ratio of the load power consumption to the suppled power. The load power consumption is the consumed energy on the load resistance Ro

The resistance of receiver R_2 can be expressed as this:

$$\mathbf{R_2 = R_C + Ro = R_C\,(1+ Ro/ R_C) = R_C\,(1+r)} \qquad (3.6)$$

where r is the load retio defined as this:

$$\mathbf{r= Ro/R_C}$$

Then, Q_2 can be expressed as this:

$$\mathbf{Q_2 = Q_2^{\#}/ (1+r)} \qquad (3.7)$$

where $\mathbf{Q_2^{\#}}$ is the $\mathbf{Q_2}$ value of the receiver circuit without load Ro. The transmission efficiency η is given by

$$\boldsymbol{\eta} = \frac{\mathbf{Ro\, k^2QQ_2}}{\mathbf{R_2\,(1+ k^2QQ_2)}}$$

$$= \frac{1}{\left\{\dfrac{1}{k^2QQ^{\#}_2}(1+r) +1\right\}\left(1+ \dfrac{1}{r}\right)} \qquad \mathbf{(3.8)}$$

Then, the maximum transmission efficiency η_{max} becomes:

$$\boldsymbol{\eta_{max}} = \frac{\mathbf{k^2\,QQ^{\#}_2}}{\mathbf{\{1+ \sqrt{1+k^2QQ^{\#}_2}\}^2}} \qquad \mathbf{(3.9)}$$

when the optimized load r_M is

$$\mathbf{r_M = \sqrt{1+k^2\,QQ^{\#}_2}} \qquad \mathbf{(3.10)}$$

Eq. (2.4) shows that the maximum transmission efficiency η_{max} becomes more than 95% when $\mathbf{k^2QQ^{\#}_2 > 10000.}$

If $\mathbf{k^2QQ^{\#}_2}$ >> 1 , the optimized load $\mathbf{r_M}$ can be approximated with

$$\mathbf{r_M \fallingdotseq k\sqrt{QQ^{\#}_2}} \qquad \mathbf{(3.11)}$$

Therefore, $\mathbf{Q_2}$ can be approximated with this:

$$Q_2 \fallingdotseq \frac{Q_2^{\#}}{1+k\sqrt{QQ^{\#}_2}} \qquad (3.12)$$

Further, if the transmitter coil is equals to that of receiver ($Q^*_2 = Q_1$), then,

$$Q_2 \fallingdotseq 1/k \qquad (3.13)$$

Therefore, the quality factor of the receiver should be designed to be equal or near to the inverse of coupling coefficient.

Therefore, under the optimized design,

$$k^2QQ_2 \fallingdotseq kQ \fallingdotseq Q / Q_2 = r + 1 \qquad (3.14)$$

The above results implies that as illustrated in Fig. 3.4 the problem of large electromagnetic noise of the resonant type completely solvable if the supplied current frequency can be spread among the$(\mathbf{r + 1})\Delta f$ frequency (Δf : frequency interval).

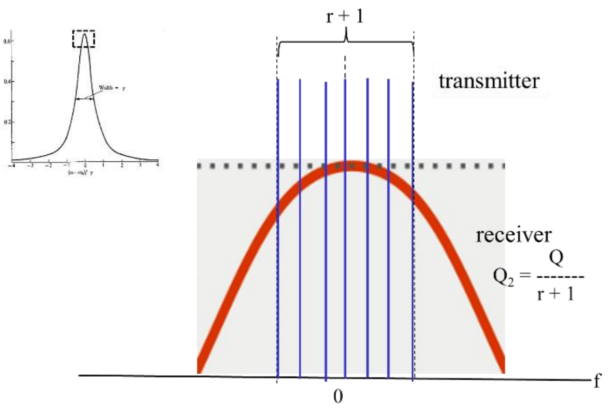

Fig. 3.4 Spread on Frequency Domain

Let the design objective of transfer efficiency be more than 95%. From Eq. (3.9), $\mathbf{k^2QQ^{\#}_2}$ becomes more than 10000. When the coupling coefficient k be equal to 0.1, $\mathbf{QQ^{\#}_2}$ should be design to be 1000000. Then Q_1 should be more than 1000 if $\mathbf{Q^{\#}_2 = Q}$. Therefore, from Eq. (3.13) the load ratio r becomes more then 100 since $\mathbf{Q_2}$ should be near $\mathbf{1/k}$

3.5 efficiency robustness

To analyze the robustness of efficiency against slight frequency shifts, this section analyzes the behavior of the resonant type near the resonant state where slight frequency shifts from the resonance frequency.

From Eq. (2.4) and Eq. (2.5), the efficiency in non-resonant states η_N becomes

$$\eta_N = \frac{Ro\, I^*_2 I_2}{V\sqrt{I^*_1 I_1}} \qquad (3.15)$$

$$= \frac{Ro\, k^2\sqrt{Q^{\wedge *}Q^{\wedge}Q^{\wedge *}_2Q^{\wedge}_2R^{\wedge *}R^{\wedge}}}{\sqrt{R^{\wedge *}R^{\wedge}R^{\wedge *}_2R^{\wedge}_2(1+k^2Q^{\wedge *}Q^{\wedge *}_2)(1+k^2Q^{\wedge}Q^{\wedge}_2)}}$$

Where '*' denotes the complex conjugates. Under the optimize design,

$$Q_1 >> Q_2 \fallingdotseq 1/k \qquad (3.16)$$

Therefore, the receiver circuit can be assumed to be in the ideal resonant state for the receiver circuit. Then the efficiency in the half-resonant states η_H can be simplified as this:

$$\eta_H = \frac{Ro\, k^2 Q\sqrt{Q^{\wedge *}_2Q^{\wedge}_2}}{\sqrt{R^{\wedge *}_2R^{\wedge}_2(1+k^2Q^*Q^{\wedge *}_2)(1+k^2QQ^{\wedge}_2)}}. \qquad (3.17)$$

In the calculation of the efficiency in the half-resonant states η_H, two parts are divided. By substituting Eq. (2.7) and Eq. (2.9), the fist factor becomes

$$\frac{Q^{\wedge *}_2Q^{\wedge}_2}{R^{\wedge *}_2R^{\wedge}_2} = \frac{Q_2\,Q_2}{(1-jQ_2\Omega_2)(1+jQ_2\Omega_2)}.$$

$$\times \frac{1}{R_2\,R_2\,(1-jQ_2\Omega_2)(1+jQ_2\Omega_2)}.$$

$$= \frac{Q_2^2}{R_2^{\,2}}\left\{\frac{1}{\{(1+(Q_2\Omega_2)^2\}}\right\}^2. \qquad (3.18)$$

By substituting Eq. (2.7) and Eq. (2.9), the second factor becomes

$$(1+k^2QQ^{\wedge *}_2)(1+k^2QQ^{\wedge}_2) \qquad (3.19)$$

$$= \left\{1+\frac{k^2QQ_2}{(1-jQ_2\Omega_2)}\right\}\left\{1+\frac{k^2QQ_2}{(1+jQ_2\Omega_2)}\right\}$$

$$= (1+k^2QQ_2)^2 - (2k^2QQ+k^4Q^2Q_2^2)\frac{(Q_2\Omega_2)^2}{\{1+(Q_2\Omega_2)^2\}}$$

Near the resonant state (at least $\mathbf{Q_2\Omega_2 < 0.5}$), $\mathbf{Q_2\Omega_2} \to 0$. Therefore, the second factor asymptotes like this:

$$(1+k^2QQ^{\wedge *}_2)(1+k^2QQ^{\wedge}_2) \to (1+k^2QQ_2)^2 \qquad (3.20)$$

Therefore, the efficiency near the resonant state η_N can be approximated as this:

$$\eta \to \frac{Ro\, k^2QQ_2}{R_2\,(1+k^2QQ_2)}\ \frac{1}{\{1+(Q_2\Omega_2)^2\}}. \qquad (3.21)$$

In Eq. (3.21), the rear part is equal to Lorentz function without constants. Therefore, the above results show that the efficiency near the resonant state behave like a resonance curve.

3.6 robustness criterion for resonances

This section proposes a robustness criterion for the resonances and the Q value.

To obtain the engineering criterion, this paper considers the discrepancy from the ideal resonance as a kind of noise. Generally, noises are estimated with S/N retio (signal to noise retio). The resonances can be chracterized with Lorentz function in which the frequency of power source ω is not equal to the natural frequency of systems. Therefore, to enable noises estimation with S/N retio the Lorentz function is expand to the Taylor series.

Let $\boldsymbol{\omega}$ be angular frequency, $\boldsymbol{\omega_0}$ be the natural frequency of a system and $\boldsymbol{\gamma}$ be the full width half maximum (FWHM). Then, the Lorentz function charactering the resonances is

$$\boldsymbol{\Gamma(x) = \frac{1}{\pi\ (\gamma/2)\ (1+x^2)}} \qquad \textbf{(3.22)}$$

where

$$\mathbf{x = \frac{\omega - \omega_0}{(\gamma/2)}}. \qquad \textbf{(3.23)}$$

The discrepancy from the ideal resonance increases with the increasing x from zero in which the frequency of power source ω is equal to the natural frequency of systems. Under the ideal resonance the circuit, then Lorentz function becomes

$$\boldsymbol{\Gamma(0) = \frac{1}{\pi\ (\gamma/2)}}\ . \qquad \textbf{(3.24)}$$

Them, the Taylor expansion around zero is

$$\boldsymbol{\Gamma(x) = \frac{1}{\pi\ (\gamma/2)}\{\ 1 - x^2 + x^4 - \ldots\ \}}\ . \qquad \textbf{(3.25)}$$

By considering the discrepancy as a kind of noise, the second term in Eq. (3.25) is regurded as the noise source. Then, S/N retio can be written

$$\mathbf{(S/N)^2 = (x^2)^2}. \qquad \textbf{(3.26)}$$

Let the allowance be less than -20 dB (energy losses is less than 1%). Then, the range of x that satisfies the allowance is

$$\mathbf{-0.1 < x^2 < 0.01}\ . \qquad (3.27)$$

By substituting Eq. (3.2) to Eq. (3.6), the range of the frequency $\boldsymbol{\omega}$ that is less than -20 dB becomes

$$\mathbf{-0.1\ (\gamma/2) < \omega - \omega_0 < 0.1\ (\gamma/2)}\ . \qquad (3.28)$$

The above relations implies that the allowance of frequency shift are within 10% of the FWHM the under resonances. For this reason, in this paper circuits and phenomena that satisfies the engineering criterion less than -20 dB are called ‘pseudo-resonance.’

Base on the discussions so far, the problem of large electromagnetic noise of the resonant type completely solvable if the **(r + 1)** frequency can be spread within 10% of the FWHM without significant loss of efficiency. Therefore, this paper concludes that the spread spectrum is one of promising technique for noise suppression of the resonant type

4. CONCLUSION

This paper theoretically clarifies for effectiveness of the spread spectrum [3] for noise suppression on the resonant type WPT systems [1]. For the resonant type, slight frequency shift are equivalent to the virtually parallel transmission on the frequency domain. Therefore, it enables theoretically us to transmit energy without significant loss of efficiency. Implementation challenges such as stability of oscillation, suppression of interferences among spread frequencies are for further studies.

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